Letters

Comments on "Time-Domain Reflectometry Using Arbitrary Incident Waveforms"

Jacek Izydorczyk

This paper suggests slight changes to the time-domain reflectometry algorithm presented in the above paper.[1] Proposed changes make the algorithm simpler to implement. An example of computations is given.

Recently, I found excellent text on time-domain reflectometry published in the above paper. While examining the references, I found that the ideas described in the above paper probably evolved from some earlier concepts published in [1]. The main idea of both papers is very clever and I admire the authors for their ingenuity, but I think that the published algorithm, concerning the impedance profile identification of the nonuniform lossless transmission line (NLTL), is unnecessarily complicated. I suggest below slight changes to the algorithm that make it possible to be implemented even on a programmable calculator.

The authors of the above paper consider inverse scattering problem. Scattering media is an NLTL approximated by a cascaded N-section uniform line (see Fig. 1). It is assumed that the line excitation $u^+(t)$ can have an almost arbitrary shape. The goal is to identify the impedance profile of the transmission line—consequently, the "shape" of the line—by detection of the reflected wave $u^-(t)$. It is known [2] that, if the spectrum of the incident wave $U^+(j\omega)$ is nonzero only for angular frequencies $\omega \in \langle -\pi/2\tau; \pi/2\tau \rangle$, where $\tau = \Delta\ell/c$ is a propagation delay across one taper of the line, then incident and reflected waves can be exactly described by their sampled versions $u^+[n] = u^+(n \cdot 2\tau)$ and $u^-[n] = u^-(n \cdot 2\tau)$. As a result, we can use the reflection coefficient of the NLTL in the $\mathcal{Z}$-domain

$$D(z) = \frac{\sum_{n=0}^{\infty} u^-[n] \cdot z^{-n}}{\sum_{n=0}^{\infty} u^+[n] \cdot z^{-n}} \tag{1}$$

where $z = \exp(j\omega \cdot 2\tau)$. If the NLTL consists of N cascaded uniform lines, then the reflection coefficient is a rational function of the z^{-1} variable

$$D(z) = \frac{\sum_{n=0}^{N} b_n \cdot z^{-n}}{\sum_{n=0}^{N} a_n \cdot z^{-n}} = \frac{A(z^{-1})}{B(z^{-1})}. \tag{2}$$

The order of the reflection coefficient N is equal to the number of cascaded tapers. Polynomials $A(z^{-1})$ and $B(z^{-1})$ can be evaluated by a recursive formula. Incident wave $U_n^+(z)$ and reflected wave $U_n^-(z)$ at the entrance of the nth section of the NLTL are related to the incident and reflected waves $U_{n+1}^+(z), U_{n+1}^1(z)$ at the entrance of the $(n+1)$st section by a transmission matrix [2]

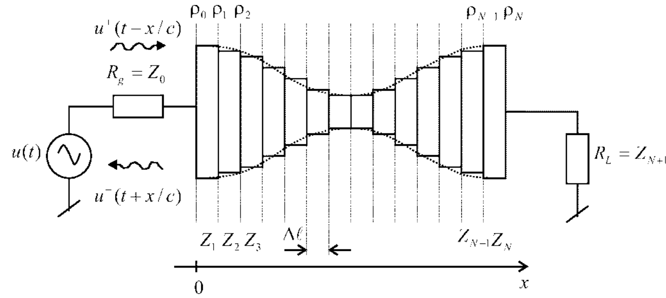

Fig. 1. NLTL approximated by cascaded N-segments of a ULTL.

$$\begin{aligned}\begin{bmatrix} U_n^+(z) \\ U_n^-(z) \end{bmatrix} &= \frac{1}{1+\rho_n} \begin{bmatrix} 1 & \rho_n z^{-1} \\ \rho_n & z^{-1} \end{bmatrix} \begin{bmatrix} U_{n+1}^+(z) \\ U_{n+1}^-(z) \end{bmatrix} \\ &= \mathbf{T}_n(z) \begin{bmatrix} U_{n+1}^+(z) \\ U_{n+1}^-(z) \end{bmatrix}\end{aligned} \tag{3}$$

where ρ_n is a reflection coefficient

$$\rho_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}, \quad n = 0, 1, \ldots, N. \tag{4}$$

The transmission matrix $\mathbf{T}_n(z)$ describes reflective (and transmission) properties of the junction between two sections of a uniform lossless transmission line (ULTL). The transmission matrix of two cascaded junctions (a model of the NLTL consists of only one segment of a ULTL in this case) is equal to the product of the transmission matrices of all junctions. For example, for the first and second junction, the transmission matrix takes the form

$$\mathbf{T}_{0,\ldots,1}(z) = \mathbf{T}_0(z)\mathbf{T}_1(z) = \frac{\begin{bmatrix} 1 + \rho_0\rho_1 z^{-1} & \rho_1 z^{-1} + \rho_0 z^{-2} \\ \rho_0 + \rho_1 z^{-1} & \rho_0\rho_1 z^{-1} + z^{-2} \end{bmatrix}}{(1+\rho_0)\cdot(1+\rho_1)}. \tag{5}$$

Equations (3) and (5) suggest that the transmission matrix for the n-cascaded ULTL has the form

$$\mathbf{T}_{0,\ldots,n}(z) = \begin{bmatrix} T_{11}^{(n)}(z^{-1}) & T_{12}^{(n)}(z^{-1}) \\ T_{21}^{(n)}(z^{-1}) & T_{22}^{(n)}(z^{-1}) \end{bmatrix} = \begin{bmatrix} B_n(z^{-1}) & A_n^R(z^{-1}) \\ A_n(z^{-1}) & B_n^R(z^{-1}) \end{bmatrix} \tag{6}$$

where $A_n(z^{-1})$ and $B_n(z^{-1})$ are polynomials of the z^{-1} variable. Polynomial $A_n^R(z^{-1})$ is obtained from polynomial $A_n(z^{-1})$ by reversing the order of coefficients

$$A_n^R(z^{-1}) = z^{-(n+1)} \cdot A_n(z). \tag{7}$$

Equation (7) means that the order of the polynomial $A_n^R(z^{-1})$ is greater then the order of the polynomial $A_n(z^{-1})$ by one. Polynomial $B_n(z^{-1})$ is converted into $B_n^R(z^{-1})$ in a similar way. Proof of (6) can be done by induction. For $n = 1$, the validity of (6) is evident by

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[1] T.-W. Pan, C.-W. Hsue, J.-F. Huang, *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 11, pp. 2558–2563, Nov. 2002.

inspection of (5). If (6) is true for n sections, then the transmission matrix for $n+1$ sections takes the form

$$\mathbf{T}_{0,\ldots,n+1}(z) = \begin{bmatrix} A_n & B_n^R \\ B_n & A_n^R \end{bmatrix} \cdot \frac{\begin{bmatrix} 1 & \rho_{n+1} z^{-1} \\ \rho_{n+1} & z^{-1} \end{bmatrix}}{1+\rho_{n+1}} = \frac{\begin{bmatrix} A_n + \rho_{n+1} B_n^R & A_n \rho_{n+1} z^{-1} + B_n^R z^{-1} \\ B_n + \rho_{n+1} A_n^R & B_n \rho_{n+1} z^{-1} + A_n^R z^{-1} \end{bmatrix}}{1+\rho_{n+1}}. \tag{8}$$

Diagonal elements of $\mathbf{T}_{0,\ldots,n+1}(z)$ are polynomials related by the "reversion" rule

$$\begin{aligned}\left(T_{11}^{(n+1)}(z^{-1})\right)^R &= z^{-(n+2)} \cdot \left(A_n(z) + \rho_{n+1} B_n^R(z)\right) \\ &= z^{-1} A_n^R(z^{-1}) + z^{-1} \rho_{n+1} B_n(z^{-1}) \\ &= T_{22}^{(n+1)}(z).\end{aligned} \tag{9}$$

The out-of-diagonal elements are related in a similar way as follows:

$$\begin{aligned}\left(T_{21}^{(n+1)}(z^{-1})\right)^R &= z^{-(n+2)} \cdot \left(B_n(z) + \rho_{n+1} A_n^R(z)\right) \\ &= z^{-1} B_n^R(z^{-1}) + z^{-1} \rho_{n+1} A_n(z^{-1}) \\ &= T_{12}^{(n+1)}(z).\end{aligned} \tag{10}$$

Equations (9) and (10) complete the proof.

A different proof of (6) can be found in [2].

The reflection coefficient $D(z)$ of the NLTL can be derived from the transmission matrix $\mathbf{T}_{0,\ldots,N}(z)$. The termination resistance R_L can be interpreted as the impedance of the $(N+1)$st semi-infinite ULTL. Due to the lack of impedance variations inside the last ULTL, we do not observe the reflected wave $U_{N+1}^-(z) = 0$

$$\begin{bmatrix} U^+(z) \\ U^-(z) \end{bmatrix} = \begin{bmatrix} U_0^+(z) \\ U_0^-(z) \end{bmatrix} = \begin{bmatrix} B_N(z^{-1}) & A_N^R(z^{-1}) \\ A_N(z^{-1}) & B_N^R(z^{-1}) \end{bmatrix} \begin{bmatrix} U_{N+1}^+(z) \\ 0 \end{bmatrix}. \tag{11}$$

As a result, the reflection coefficient $D(z)$ equals

$$D(z) = \frac{U^-(z)}{U^+(z)} = \frac{A_N(z^{-1})}{B_N(z^{-1})}. \tag{12}$$

If we omit the constant factor, then the numerator $A_N(z^{-1})$ is equal to the polynomial $A(z^{-1})$ from (2), and the denominator $B_n(z^{-1})$ is equal to the polynomial $B(z^{-1})$.

Equation (8) shows how to compute polynomials $A_n(z^{-1})$ and $B_n(z^{-1})$ if we know polynomials $A_{n-1}(z^{-1})$ and $B_{n-1}(z^{-1})$ and the reflection coefficient ρ_n

$$\begin{bmatrix} A_n \\ B_n^R \end{bmatrix} = \begin{bmatrix} 1 & \rho_n \\ \rho_n z^{-1} & z^{-1} \end{bmatrix} \begin{bmatrix} A_{n-1} \\ B_{n-1}^R \end{bmatrix}. \tag{13}$$

Using (13) iteratively, we can compute polynomials $A_N(z^{-1})$ and $B_N(z^{-1})$ from reflection coefficients $\rho_0, \rho_1, \ldots, \rho_N$ starting from

$$A_0(z^{-1}) = 0 \text{ and } B_0(z^{-1}) = 1. \tag{14}$$

Each step of the iteration can be interpreted as the addition of a new segment of the ULTL at the end of the NTTL.

It can be easily proven [2] that all polynomials $B_n(z^{-1})$ obtained by means of (13) with initial conditions (14) are monic, i.e., $\forall_{n=0,1,2,\ldots}, B_n(0) = 1$.

At this point, we can use an idea from the above paper and [1], which says that the reflected wave in the time instant $t_{n+1} = (n+1)\cdot 2\tau$ consists of the waves reflected from junctions created by the first n-sections of the NLTL and a small component, which has traveled from the source to the junction between nth and $(n+1)$st sections and has returned to the source by the shortest way as follows:

$$u^-[n+1] = \sum_{k=0}^{n} a_k^{(n)} u^+[n-k] - \sum_{k=1}^{n} b_k^{(n)} u^-[n-k] + u^+[0]\rho_{n+1} \prod_{k=1}^{n} (1-\rho_k^2). \tag{15}$$

In (15), coefficients of the polynomials $A_n(z^{-1})$ and $B_n(z^{-1})$ are $a_0^{(n)}, a_1^{(n)}, \ldots, a_n^{(n)}$ and $b_0^{(n)}, b_1^{(n)}, \ldots, b_n^{(n)}$, respectively. If we know the first n reflection coefficients $\rho_0, \rho_1, \ldots, \rho_n$, then we can compute the coefficients of $A_n(z^{-1})$ and $B_n(z^{-1})$ polynomials. Furthermore, ρ_{n+1} can be evaluated from (15) using the first $n+1$ samples of incident and reflected waves. If we know ρ_{n+1}, then we can compute, using (13), coefficients of polynomials $A_{n+1}(z^{-1})$ and $B_{n+1}(z^{-1})$. The next sample of reflected wave can be used to compute ρ_{n+2} in a similar manner. If we know the reflection coefficients of the junctions, it is a simple task to compute the wave impedances

$$Z_{n+1} = Z_n \cdot \frac{1+\rho_n}{1-\rho_n}, \quad n = 0, 1, \ldots, N \tag{16}$$

where $Z_0 = R_k$.

The algorithm can be represented as the following piece of pseudocode:

```
r[0] = u⁻[0]/u⁺[0];
a[0] = r[0];
b[0] = 1;
Z[0] = Z0;
z[1] = Z[0] * (1−r[0])/(1+r[0]);
for (n = 1; n < N; n++)
 {
 du = Σ_{k=0}^{n−1} a[k]u⁺[n−k] − Σ_{k=1}^{n} b[k]u⁻[n−k]−u⁻[n];
 r[n] = du/u⁺[0]/ Π_{k=0}^{n−1}(1−r[k] * r[k]);
 aux1 = reverse(b, n+1)+r[n] * a
 aux2 = a+r[n] * reverse(a, n+1);
 b = aux1;
 a = aux2;
 Z[n+1] = Z[n] * (1+r[n])/(1−r[n]);
 }
```

Sampled incident and reflected waves are stored in arrays $\mathbf{u}^+[\mathbf{N}]$ and $\mathbf{u}^-[\mathbf{N}]$, respectively. Computed reflection coefficients of junctions are stored in the array $\mathbf{r}[\mathbf{N}]$. Coefficients of polynomials $A_n(z^{-1})$ and $B_n(z^{-1})$ are stored in the arrays $\mathbf{a}[\mathbf{N}]$ and $\mathbf{b}[\mathbf{N}]$. Computed impedances are written to array $\mathbf{Z}[\mathbf{N}]$. Function $\mathbf{reverse}(\mathbf{x}, \mathbf{N})$ returns array $\mathbf{x}$ with reversed order of first $\mathbf{N}+\mathbf{1}$ elements. Constant $\mathbf{Z0}$ is equal to source resistance R_g.

In the algorithm, we omit tedious enumeration of all possible reflections from junctions between ULTL sections, which is a main goal of the algorithm depicted in Fig. 4 of the above paper.

The best way to test the proposed algorithm is to use data from measurements conducted by the authors of the above paper on a variable-strip microstrip line. Since such sort of a test is not possible for

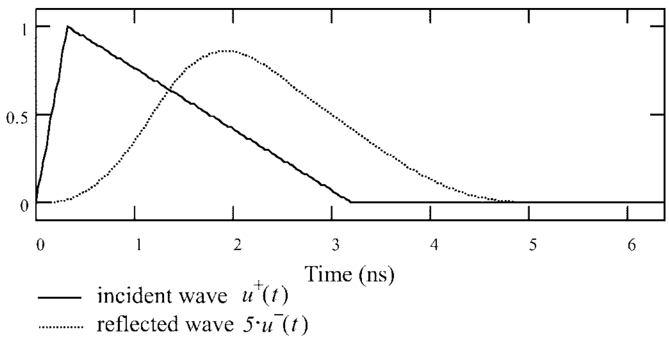

Fig. 2. Wave incident to triangular taper and reflected wave.

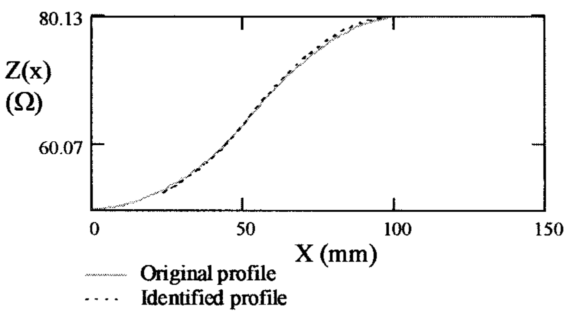

Fig. 3. Original and identified impedance profile of triangular taper.

me, only simulation results will be presented. Triangular taper from the above paper have been used for the simulation. Impedance profile of the taper is as follows:

$$Z(z) = \begin{cases} R_g \cdot \exp\left(2\left(\frac{x}{L}\right)^2 \cdot \ln\left(\frac{R_L}{R_g}\right)\right), & \text{if } 0 \le x < \frac{L}{2} \\ R_g \cdot \exp\left(\left(4\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2 - 1\right) \cdot \ln\left(\frac{R_L}{R_g}\right)\right), & \\ \quad \text{if } \frac{L}{2} \le x \le L. & \end{cases} \tag{17}$$

The source resistance $R_g = 50\ \Omega$, load resistance $R_g = 80\ \Omega$, length $L = 10$ cm, and velocity of electromagnetic waves $c = 10^8$ m/s. The incident wave has the form of a nonsymmetrical triangular wave with a rise time from 0 to 1 V equal to 0.325 ns and a fall time from 1 V to 0 equal to 2.875 ns. Duration of the incident wave is longer then the time needed to travel across the taper, reflect from the loading resistance, and return to source. The algorithm described in [3] has been used to simulate a reflection coefficient of the NLTL. In order to compute the reflected wave, fast Fourier transform (FFT)-based convolution [4] has been used (see Fig. 2). Simulated and identified impedance profiles are compared in Fig. 3. The reconstruction accuracy is similar to accuracy reported in the above paper.

References

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Authors' Reply

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We gratefully acknowledge Mr. Izydorczyk's comments on the above paper.[1] Mr. Izydorczyk derives the reflection coefficient $D(z)$ of the nonlinear transmission line (NLTL) by using the transmission matrix method, which is not found in the above paper. In addition, by omitting tedious enumeration of all possible transmission-reflection processes from junctions between uniform lossless transmission line (ULTL) sections, the text presents a simple algorithm to compute a wave impedance profile of the NLTL. We think the proposed algorithm is simple and accurate enough for some practical applications. In particular, Mr. Izydorczyk's comments further proves that an incident wave having the duration of pulsewidth longer than the time needed for the wave to travel across the NLTL can be employed to characterize the impedance profile of an NLTL.

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[1] T.-W. Pan, C.-W. Hsue, and J.-F. Huang, *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 11, pp. 2558–2563, Nov. 2002.

Corrections to "Design of an Interdigital Hairpin Bandpass Filter Utilizing a Model of Coupled Slots"

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The above paper[1] contains an error in (6). The correct form of (6) is as follows:

$$\left.\begin{aligned} P^k &= \frac{1}{2}\left(Vm_k^T\right)^* Im_k \\ 0 &= Vm_l^T Im_m \ l \neq m \end{aligned}\right\}, \qquad k, l, m = 1, \ldots, n. \tag{6}$$

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[1] A. N. Deleniv, M. S. Gashinova, I. B. Vendik, and A. Eriksson, *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 9, pp. 2153–2158, Sept. 2002.